

## Dynamic Regression Models Used in Agriculture

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### SUMMARY

Regression modelling concerns the construction of a mathematical and statistical description of the effect of independent or regressor variables on the response time series  $Y_t$ . Economic variables are liable to follow slowly-evolving trends and they are also liable to be strongly correlated with each other. If the disturbance term is indeed compounded from such variables, then we should expect that it too will follow a slowly-evolving trend.

### INTRODUCTION

Regression models are widely used in quantitative analyses. However, for the geographer they can be problematic because, among a range of assumptions, they are required to be independent of the residuals. Two types of time series models are Static / Instantaneous model and Multiperiod model. Static Model is postulated when a change in  $X$  at time  $t$  is believed to have an immediate effect on  $Y$ .  $Y_t = \alpha + \beta X_t + U_t$  Multiperiod models would contain variables of different time denomination.

They are also known as dynamic models.  $Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + U_t$  Dynamic model involves lagged variables and Lag is the lapse of time. In economics, realistic formulations of economic relations often require the insertion of lagged values of the explanatory variables. Examples: i. Supply depends on lagged prices ii. Demand for durable goods. iii. Bank money and reserve ratio IV. Inflation and money supply, interest rate cut.

### Types of lags found in the implementation of macro-economic policy:

- **Data lags**-Many macroeconomic data series such as GDP are only available with a considerable lag, because of this getting information about the current state of the economy is difficult.
- **Recognition lag**-Once the data are finally available it takes time to figure out what they are saying, if it's temporary then no action needed but if permanent then action may be needed.
- **Legislative lag**-Once the necessary data is obtained and concluded then something is needed to be done but sometimes there can be considerable lags in the legislative processes.
- **Implementation lag**-Once a policy is passed, it takes time to put in place.
- **Effectiveness lag**-After the policy is put into place, it takes time for the policy to hit the economy and take effect.

### Types of Dynamic Models:

#### Pure Autoregressive Models (AR):

A model that consists of one single variable in different time periods. It includes one or more past values of the dependent variable among its explanatory variable.

Simple AR model:  $Y_t = \alpha + \beta_1 Y_{t-1} + U_t$

Higher order AR model:  $Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + U_t$

#### Pure Distributed Lag Regression Model (DL) :

A model that consists of lagged variables as independent variables.

Simple DL model:  $Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + U_t$

Higher order DL model:  $Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \dots + \beta_k X_{t-k} + U_t$

#### Pure Moving Average Model (MA) :

A model that consists of lagged error variables as independent variables.

Simple MA model:  $Y_t = \alpha + \beta_0 U_t + \beta_1 U_{t-1} + V_t$

Higher order MA model:  $Y_t = \alpha + \beta_0 U_t + \beta_1 U_{t-1} + \beta_2 U_{t-2} + \dots + \beta_p U_{t-q} + V_t$

**Mixed Models**

a. AR-MA models:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \gamma_0 U_t + \gamma_1 U_{t-1} + \gamma_2 U_{t-2} + \dots + \gamma_p U_{t-q} + V_t$$

b. AR-DL models:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \alpha_0 X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha_q X_{t-q} + V_t$$

**Distributed Lag Model:**

In regression analysis involving time series data if model include the current but also past values of explanatory variable it is called a distributed lag model. The general form of distributed lag model with k period lag is

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \dots + \beta_k X_{t-k} + U_t$$

where, Individual coefficients  $\beta$ 's are called lag weights and the collectively comprise the lag distribution,  $\beta_0$  -Impact multiplier (short run multiplier),  $\beta_t$ -Intermediate / interim multiplier of order t,  $\sum \beta_k$  - Long run multiplier or total distributed lag multiplier.

The distributed lag model can also be written as

$$Y_t = \alpha + \sum_{k=0}^p \beta_k X_{t-k} + U_t$$

**Estimation of DL model:**

There are two Methods of estimation

- a. Adhoc Estimation Method
- b. Aproiri Approach

**Adhoc Estimation Method:**

This method of estimation was given by Alt and Tinbergen

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + U_t \text{ (one lag)}$$

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + U_t \text{ (two lag)}$$

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + U_t \text{ (three lag)}$$

Sequential procedure continues till  $\beta_i$  is found insignificant or changes sign.

**Example:**

$$Y_t = 8.27 + 0.171 X_t$$

$$Y_t = 8.27 + 0.111 X_t + 0.064 X_{t-1}$$

$$Y_t = 8.27 + 0.111 X_t + 0.064 X_{t-1} - 0.005 X_{t-2}$$

In above example the sign of coefficient of  $X_{t-2}$  changes, thus the sequential procedure stops at lag 2.

**Drawbacks:**

- There is no a priori guide to decide what is the maximum length of lag.
- As we use successive lags, one is left with fewer degrees of freedom (n-k). One observation is lost and the burden of one more coefficient increases because addition of one lag variable make us loose one sample point and the sample size is reduced.  
 $(n-1)-(k+1) = n-k-2$  { additional coefficient & reduced sample size }  
 $(n-2)-(k+2) = n-k-4$
- Successive values of explanatory lag variables are highly correlated leading to multicollinearity problem. Eg:  $X_t, X_{t-1}$  and  $X_{t-2}$  are same variables.
- Lag selection is purely based on statistical significance, and so these procedures are not based on theoretical considerations, exploratory and are accused of data mining.

**Apriori Restrictions Method:**

In this method prior restrictions are given on the distributed lag model coefficients. Pattern of regression coefficients is prior assumed.

Two types of apriori methods :

- Koyck's Approach
- Almon's Approach

**Koyck's Approach:**

It was given by L.M. Koyck. This method assumes that effect/ impact decreases with time and it decreases geometrically. Thus, Koyckmodel is a geometric lag model:  $\beta_i = \beta_0 \lambda^i$ ,  $i = 1, 2, 3, \dots, \infty$ . In this figure,  $\lambda$  is the rate of decline/decay ( $0 < \lambda < 1$ ) and  $1 - \lambda$  is the speed of adjustment. We can see that as lag increases the impact decreases.

Long term impact in the distributed lag model is

$$\begin{aligned} \sum \beta_i &= \beta_0 + \beta_1 + \beta_2 + \dots \\ &= \beta_0 + \beta_0 \lambda + \beta_0 \lambda^2 + \dots \\ &= \beta_0 (1 + \lambda + \lambda^2 + \dots) \end{aligned}$$

$$\sum \beta_i = \beta_0 [1 / (1 - \lambda)] \quad (\text{geometric series})$$

- Median lag -It is the time required for 50% of the total change in Y following a unit sustained change in X. Koyck's Median lag =  $\log_2 \log \lambda$
  - Mean lag -It is the weighted average of all the lags involved, with the respective  $\beta$  coefficients serving as weights. Koyck's Mean lag =  $\lambda / (1 - \lambda)$
  - Both median and mean lag would characterize the speed of change in distributed lag models.
  - With the estimation of  $\alpha$ ,  $\beta_0$  and  $\lambda$ , we could estimate the entire regression coefficients.
  - In this method of estimation we would lose only one data point in a sample and prevents the lose of additional degrees of freedom.
  - Thus, the infinite distributed lag model was converted into simple autoregressive model.
- DL  $\rightarrow$  AR [KOYCK's Transformation]

**Drawback:**

- The inclusion of  $Y_{t-1}$  may lead to biased estimation if  $Y_{t-1}$  and  $U_t$  are correlated.
- $V_t$  will be autocorrelated even if  $U_t$  is not auto correlated because  $V_t = U_t - \lambda U_{t-1}$

**Example:**

Let us take a consumption function

$$\begin{aligned} C_t &= -38.11 + 0.52 YD_t + 0.46 C_{t-1} & R^2 &= 0.998 \\ \text{S.E.} & & & \\ & (0.12) & (0.12) & \\ \text{t-ratios} & [4.44] & [3.74] & \end{aligned}$$

**Sol:**

In the above given equation,

$$\lambda = 0.46 \text{ and } \beta_0 = 0.52 \text{ thus, } \alpha = \alpha * 1 - \lambda = -38.11 / 1 - 0.46 = -70.57$$

$$\text{Mean lag} = \lambda / (1 - \lambda) = 0.46 / (1 - 0.46) = 0.85 \quad \text{Median lag} = \log_2 \log \lambda = \log_2 \log 0.46 = 0.8926$$

As we know,  $\beta_i = \beta_0 \lambda^i$

$$\text{Then } \beta_1 = \beta_0 \lambda = 0.52 \times 0.46 = 0.24 \quad \beta_2 = \beta_0 \lambda^2 = 0.52 \times (0.46)^2 = 0.11$$

Thus, the distributed lag model using KOYCK's approach is

$$C_t = -70.57 + 0.52 YD_t + 0.24 YD_{t-1} + 0.11 YD_{t-2} + \dots + U_t$$

Where,

$C_t$  = Current purchases of goods and services

$YD_t$  = Current levels of disposable income

$YD_{t-1}, YD_{t-2}$  = Past levels of disposable income

## CONCLUSION

One of the most widely used tools in statistical forecasting, single equation regression models for forecasting with Univariate Box-Jenkins Models. Conventional linear model assume that the data are generated by a single probability density function characterised by a single set of regression parameters. It may be desirable to estimate the regression model under the assumption data are derived from a finite mixture density function and to examine difference parameter in the model.

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